



Name : .....

Roll No. : .....

Invigilator's Signature : .....

**CS/B.Tech(OLD)/SEM-2/M-201/2013  
2013  
MATHEMATICS**

Time Allotted : 3 Hours

Full Marks : 70

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

**GROUP - A  
( Multiple Choice Type Questions )**

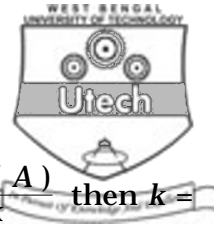
1. Choose the correct alternatives for any *ten* of the following :  
 $10 \times 1 = 10$

i)  $\frac{1}{1-D} x^2 =$

- a)  $x^2 + 2x + 1$                       b)  $x^2 + 2x$   
c)  $x^2 - 2x + 1$                       d)  $x^2 + 2x + 2.$

ii) The value of  $\begin{bmatrix} 1 & 1 & -ac & bc \\ 1 & 1 & +ad & bd \\ 1 & 1 & +ae & be \end{bmatrix} =$

- a) 1                                      b)  $abcd$   
c) 0                                      d) none of these.



iii) If  $A = \begin{bmatrix} 3 & -2 & 4 \\ 1 & 2 & -1 \\ 0 & 1 & 1 \end{bmatrix}$  and  $A^{-1} = \frac{\text{adj}(A)}{k}$  then  $k =$

- a) 15
- b) 16
- c) 0
- d) 1.

iv) The eigenvalues of  $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$  are

- a) 1, 1
- b) 6, 1
- c) 1, 0
- d) 0, 6.

v) If  $A$  is an orthogonal matrix then  $\det(A) =$

- a) 1
- b) -1
- c)  $\pm 1$
- d) 0.

vi) Integrating factor of  $x \frac{dy}{dx} + y = \log x$  is

- a)  $e^x$
- b)  $x$
- c)  $\log x$
- d) none of these.



vii) Order and degree of the differential equation  $\frac{d^2y}{dx^2} = \sqrt{1 + \frac{dy}{dx}}$  are

- a) 2, 2    b) 2, 1  
 c) 1, 2    d) 1, 1.

viii) The Wronskian of the functions  $\cos 2x$  and  $\sin 2x$  is

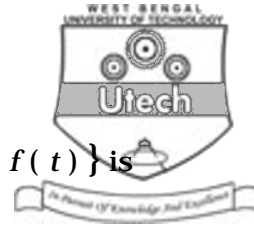
- a) 1    b) 2  
 c) - 2    d) none of these.

ix) The vectors  $(1, 1, 0, 0)$ ,  $(1, 0, 0, 1)$ ,  $(1, 0, a, 0)$ ,  $(0, 1, a, b)$  are linearly independent if

- a)  $a \neq 0$ ,  $b \neq 2$                               b)  $a \neq 2$ ,  $b \neq 0$   
 c)  $a \neq 0$ ,  $b \neq -2$                             d)  $a \neq -2$ ,  $b \neq 0$ .

x)  $T : R^2$  is defined by  $T(x, y) = (2x - y, x + y)$  then kernel of  $T$  is

- a)  $\{(1, 2)\}$                                         b)  $\{(1, -1)\}$   
 c)  $\{(0, 0)\}$                                         d)  $\{(1, 2)\}, \{(1, -1)\}$ .



xi) If  $L\{f(t)\} = \tan^{-1}\left(\frac{1}{p}\right)$  then  $L\{t, f(t)\}$  is

- a)  $\tan^{-1}\left(\frac{1}{p^2}\right)$       b)  $\frac{1}{1+p^2}$   
 c)  $\frac{1}{1+p}$       d)  $\tan^{-1}\left(\frac{2}{\pi p}\right)$ .

xii)  $(\Delta - \nabla) x^2$  is equal to

- a)  $h^2$       b)  $-2h^2$   
 c)  $2h^2$       d) none of these,

where  $h$  is equal interval.

xiii) If  $E_a$  is the absolute error in a numerical calculation whose true and approximate values are  $X_t$  and  $X_a$  then the relative error is given by

- a)  $\left| \frac{E_a}{X_a} \right|$       b)  $\left| \frac{E_a}{X_t} \right|$   
 c)  $\left| \frac{E_a}{X_t - X_a} \right|$       d)  $| E_a |$ .

**GROUP - B**

**( Short Answer Type Questions )**

Answer any *three* of the following.       $3 \times 5 = 15$

2. Examine whether the transformation  $T : R^2 \rightarrow R^2$  defined by  $T(x, y) = (2x - y, x)$  is linear or not.
3. Apply convolution theorem to find Inverse Laplace Transform of  $\frac{s}{(s^2 + 9)^2}$ .



4. Prove ( without expanding ) that

$$\begin{vmatrix} 1+a & 1 & 1 & 1 \\ 1 & 1+b & 1 & 1 \\ 1 & 1 & 1+c & 1 \\ 1 & 1 & 1 & 1+d \end{vmatrix} = abcd \left( 1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} \right).$$

5. If  $P_n ( x )$  is the Legendre's polynomial or Legendre's function of 1st kind then prove the following :

a)  $( 2n + 1 ) xP_n = ( n + 1 ) P_{n+1} + nP_{n-1}$

b)  $( 2n + 1 ) P_n = P'_{n+1} - P'_{n-1}$  where  $P'_{n+1}$  denotes the derivative of  $P_{n+1}$

6. Find the inverse of the matrix  $A = \begin{bmatrix} 2 & -3 & 4 \\ 1 & 0 & 1 \\ 0 & -1 & 4 \end{bmatrix}$

and hence solve the system :  $2x - 2y + 4z = -4$

$$x + z = 0$$

$$4z - y = 2$$

7. Evaluate  $\int_0^1 \frac{dx}{1+x}$  using Trapezoidal Rule, taking four equal sub-intervals.



**GROUP - C**

**( Long Answer Type Questions )**

Answer any *three* of the following.  $3 \times 15 = 45$

8. a) Solve the following differential equation by Variation of Parameter Method :

$$(D^2 + 1) y = \sec x \tan x$$

- b) Solve the following system using Cramer's rule :

$$x + y + z = 1$$

$$ax + by + cz = r$$

$$a^2x + b^2y + c^2z = r^2$$

where  $a \neq b \neq c$

- c) Find the missing data from the following table :

<b>x :</b>	- 2	- 1	0	1	2
<b>y :</b>	6	0	?	0	6

9. a) A linear transformation  $T : R^3 \rightarrow R^2$  is defined by  $T(x, y, z) = (x + y, x - z)$ . Find the Rank and Nullity.

- b) From the following table, construct the difference table and compute  $f(19)$  by Newton's Backward Interpolation formula.

<b>x :</b>	0	5	10	15	20
<b>f(x) :</b>	1.0	1.6	3.8	8.2	15.4



- c) Define the rank of a matrix. Find the rank of the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 6 & 7 & 8 \\ 2 & 6 & 9 & 12 & 15 \\ 4 & 8 & 12 & 14 & 16 \end{pmatrix}.$$

10. a) Show that  $(3, 1, -2)$ ,  $(2, 1, 4)$ ,  $(1, -1, 2)$  form a basis of  $R^3$ .

- b) Find the Laplace Transform of  $f(t) = \sin t$ ,  $0 < t < \pi$   
 $= 0$ ,  $t > \pi$

- c) Apply Simpson's  $\frac{1}{3}$  rd rule to evaluate  $\int_0^6 \frac{1}{(1+x)^2} dx$

taking six equal sub-intervals from  $[0, 6]$  and correct up to three decimal places.

11. a) Solve  $(D^3 - 2D^2 + D - 2) y = e^x + e^{-3x}$  where  $D = \frac{d}{dx}$ .

- b) Solve :  $\frac{dx}{dt} + 2x - 3y = t$   
 $\frac{dy}{dt} - 3x + 2y = e^{2t}$

- c) Find the general solution and singular solution of

$$y = px + \sin^{-1} p \text{ where } p = \frac{dy}{dx}.$$

12. a) Solve the following Cauchy-Euler homogeneous differential equation :

$$(x^2 D^2 - 3xD + 4) y = x^2 \text{ given that } y(1) = 1, y'(1) = 0 \text{ where } D = \frac{d}{dx}.$$



b) Solve :  $x \frac{dy}{dx} + y = y^2 x^3 \cos x$ .

c) Compute  $f(0.5)$  and  $f(0.9)$  from the following table :

<b>x :</b>	0	1	2	3
<b>f(x) :</b>	1	2	11	34

13. a) Find the eigenvalues and eigenvectors of the matrix

$$\begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}.$$

Hence diagonalise the above matrix.

b) If  $P_n(x)$  is Legendre's Polynomial then prove that

$$\int_0^6 P_n(x) P_m(x) dx = 0, \quad m \neq n$$

$$= \frac{2}{2n+1}, \quad m = n$$

OR

Find the Bessel's function,  $J_n(x)$  of 1st kind from the

Bessel's equation  $x^2 \frac{d^2y}{dx^2} + \frac{dy}{dx} + (x^2 - n^2) y = 0$ .

Hence prove that  $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x$ .