Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS / B.Tech (OLD) / SEM-2 / M-201 / 2013

2013

MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) $\dfrac{1}{1-D}\,x^2 =$

a) $x^2 + 2x + 1$ b) $x^2 + 2x$

c) $x^2 - 2x + 1$ d) $x^2 + 2x + 2$.

ii) The value of $\begin{vmatrix} 1 & 1-ac & bc \\ 1 & 1+ad & bd \\ 1 & 1+ae & be \end{vmatrix} =$

a) 1   b) $abcd$

c) 0   d) none of these.

iii) If $A = \begin{bmatrix} 3 & -2 & 4 \\ 1 & 2 & -1 \\ 0 & 1 & 1 \end{bmatrix}$ and $A^{-1} = \frac{adj(A)}{k}$ then $k =$

a) 15 b) 16

c) 0 d) 1.

iv) The eigenvalues of $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ are

a) 1, 1 b) 6, 1

c) 1, 0 d) 0, 6.

v) If A is an orthogonal matrix then det (A) =

a) 1 b) -1

c) ± 1 d) 0 .

vi) Integrating factor of $x \frac{dy}{dx} + y = \log x$ is

a) e^x b) x

c) $\log x$ d) none of these.

vii) Order and degree of the differential equation $\frac{d^2y}{dx^2} = \sqrt{1 + \frac{dy}{dx}}$ are

a) 2, 2 b) 2, 1

c) 1, 2 d) 1, 1.

viii) The Wronskian of the functions cos $2x$ and sin $2x$ is

a) 1 b) 2

c) – 2 d) none of these.

ix) The vectors (1, 1, 0, 0), (1, 0, 0, 1), (1, 0, a, 0), (0, 1, a, b) are linearly independent if

a) $a \neq 0, \ b \neq 2$ b) $a \neq 2, \ b \neq 0$

c) $a \neq 0, \ b \neq -2$ d) $a \neq -2, \ b \neq 0$.

x) $T : R^2$ is defined by $T(x, y) = (2x - y, x + y)$ then kernel of T is

a) $\{(1, 2)\}$ b) $\{(1, -1)\}$

c) $\{(0, 0)\}$ d) $\{(1, 2)\}, \{(1, -1)\}$.

xi) If $L\{f(t)\} = \tan^{-1}\left(\frac{1}{p}\right)$ then $L\{t, f(t)\}$ is

a) $\tan^{-1}\left(\frac{1}{p^2}\right)$ b) $\frac{1}{1+p^2}$

c) $\frac{1}{1+p}$ d) $\tan^{-1}\left(\frac{2}{\pi p}\right)$.

xii) $(\Delta - \nabla)\, x^2$ is equal to

a) h^2 b) $-2h^2$

c) $2h^2$ d) none of these,

where h is equal interval.

xiii) If E_a is the absolute error in a numerical calculation whose true and approximate values are X_t and X_a then the relative error is given by

a) $\left|\frac{E_a}{X_a}\right|$ b) $\left|\frac{E_a}{X_t}\right|$

c) $\left|\frac{E_a}{X_t - X_a}\right|$ d) $|E_a|$.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. 3 × 5 = 15

2. Examine whether the transformation $T : R^2 \to R^2$ defined by $T(x, y) = (2x - y, x)$ is linear or not.

3. Apply convolution theorem to find Inverse Laplace Transform of $\frac{s}{(s^2+9)^2}$.

4. Prove (without expanding) that

$$\begin{vmatrix} 1+a & 1 & 1 & 1 \\ 1 & 1+b & 1 & 1 \\ 1 & 1 & 1+c & 1 \\ 1 & 1 & 1 & 1+d \end{vmatrix} = abcd\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right).$$

5. If $P_n(x)$ is the Legendre's polynomial or Legendre's function of 1st kind then prove the following :

a) $(2n+1)xP_n = (n+1)P_{n+1} + nP_{n-1}$

b) $(2n+1)P_n = P'_{n+1} - P'_{n-1}$ where P'_{n+1} denotes the derivative of P_{n+1}

6. Find the inverse of the matrix $A = \begin{bmatrix} 2 & -3 & 4 \\ 1 & 0 & 1 \\ 0 & -1 & 4 \end{bmatrix}$

and hence solve the system :

$$2x - 2y + 4z = -4$$
$$x + z = 0$$
$$4z - y = 2$$

7. Evaluate $\int_0^1 \frac{dx}{1+x}$ using Trapezoidal Rule, taking four equal sub-intervals.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

8. a) Solve the following differential equation by Variation of Parameter Method :

$$\left(D^2 + 1 \right) y = \sec x \tan x$$

b) Solve the following system using Cramer's rule :

$$x + y + z = 1$$

$$ax + by + cz = r$$

$$a^2 x + b^2 y + c^2 z = r^2$$

where $a \neq b \neq c$

c) Find the missing data from the following table :

***x* :**	– 2	– 1	0	1	2
***y* :**	6	0	?	0	6

9. a) A linear transformation $T : R^3 \rightarrow R^2$ is defined by $T(x, y, z) = (x + y, x - z)$. Find the Rank and Nullity.

b) From the following table, construct the difference table and compute $f(19)$ by Newton's Backward Interpolation formula.

***x* :**	0	5	10	15	20
***f* (*x*) :**	1·0	1·6	3·8	8·2	15·4

c) Define the rank of a matrix. Find the rank of the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 6 & 7 & 8 \\ 2 & 6 & 9 & 12 & 15 \\ 4 & 8 & 12 & 14 & 16 \end{pmatrix}.$$

10. a) Show that (3, 1, – 2), (2, 1, 4), (1, – 1, 2) form a basis of R^3.

b) Find the Laplace Transform of

$$f(t) = \sin t, \quad 0 < t < \pi$$
$$= 0, \quad t > \pi$$

c) Apply Simpson's $\frac{1}{3}$ rd rule to evaluate $\int_0^6 \frac{1}{(1+x)^2}\, dx$ taking six equal sub-intervals from $[0, 6]$ and correct up to three decimal places.

11. a) Sove $\left(D^3 - 2D^2 + D - 2 \right) y = e^x + e^{-3x}$ where $D = \frac{d}{dx}$.

b) Solve :

$$\frac{dx}{dt} + 2x - 3y = t$$
$$\frac{dy}{dt} - 3x + 2y = e^{2t}$$

c) Find the general solution and singular solution of

$$y = px + \sin^{-1} p \quad \text{where } p = \frac{dy}{dx}.$$

12. a) Solve the following Cauchy-Euler homogeneous differential equation :

$\left(x^2 D^2 - 3xD + 4 \right) y = x^2$ given that $y(1) = 1$, $y'(1) = 0$ where $D = \frac{d}{dx}$.

b) Solvve : $x \frac{dy}{dx} + y = y^2 x^3 \cos x$.

c) Compute $f(0{\cdot}5)$ and $f(0{\cdot}9)$ from the following table :

x :	0	1	2	3
***f* (*x*) :**	1	2	11	34

13. a) Find the eigenvalues and eigenvectors of the matrix

$$\begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}.$$

Hence diagonalise the above matrix.

b) If $P_n(x)$ is Legendre's Polynomial then prove that

$$\int_0^6 P_n(x) P_m(x)\, dx = 0 \quad , \quad m \neq n$$

$$= \frac{2}{2n+1} \quad , \quad m = n$$

OR

Find the Bessel's function, $J_n(x)$ of 1st kind from the Bessel's equation $x^2 \frac{d^2y}{dx^2} + \frac{dy}{dx} + \left(x^2 - n^2\right) y = 0$.

Hence prove that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x$.