Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/B.Tech(OLD)/SEM-1/M-101/2010-11

2010-11

MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) Degree of homogeneity of $ax^2 + 2hxy + by^2$ is

a) 2 b) 1

c) 0 d) none of these.

ii) If $u + v = x$, $uv = y$ then $\frac{\partial(x, y)}{\partial(u, v)}$ is

a) $u - v$ b) uv

c) $u + v$ d) $\frac{u}{v}$.

iii) If $f(x, y) = x^2 + y^2$, then $f_{xy}(x, y) =$

a) 1   b) 0

c) 2   d) $x + y$.

iv) The sequence $\{(-1)^n\}$ is

a) convergent   b) oscillatory

c) divergent   d) none of these.

v) The series $\sum r^n$ is divergent if

a) $|r| > 1$ b) $r > 1$

c) $r \geq 1$ d) $r > 0$.

vi) If $y = 5x^{100} + 3$, then y_{100} is equal to

a) 100!   b) 500

c) 0   d) none of these.

vii) If $y = \sin 2x + \cos 2x$, then $(y_n)_0$ is equal to

a) $(-1)^n 2^n$ b) 2^n

c) 0   d) none of these.

viii) The value of x which makes the vector $xi + 2j + 8k$ and $2i + 3j - k$ perpenicular is

a) 0 b) – 1

c) 1 d) 2.

ix) If $\vec{\alpha} \cdot (\vec{\beta} \times \vec{\gamma}) = 0$, then the vectors $\vec{\alpha}$, $\vec{\beta}$, $\vec{\gamma}$ are

a) coplaner b) independent

c) collinear d) none of these.

x) If the radius of the sphere

$K (x^2 + y^2 + z^2) + x + 2y - 2z = 0$, is 1, then K =

a) ± 2 b) $\pm \frac{2}{3}$

c) $\pm \frac{3}{2}$ d) ± 1 .

xi) The maximum value of the directional derivative of

$d = 2xz - y^2$ at the point (1, 3, 2) is

a) $\sqrt{14}$ b) $2\sqrt{14}$

c) $2\sqrt{7}$ d) $3\sqrt{7}$.

xii) The value of

$$\int_0^a \int_{y=0}^{\frac{b}{a}\sqrt{a^2-x^2}} x^3 y \, dy \, dx \text{ is}$$

a) $\frac{b^2 a^3}{24}$ b) $\frac{b^2 a^4}{24}$

c) $\frac{b^4 a^3}{24}$ d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. If $W = \psi (2x - 3y, \; 3y - 4z, \; 4z - 2x)$, prove that

$$\frac{1}{2}\frac{\partial W}{\partial x} + \frac{1}{3}\frac{\partial W}{\partial y} + \frac{1}{4}\frac{\partial W}{\partial z} = 0.$$

3. Test the convergence of the series $\sum \frac{1}{\sqrt{n}} \sin \frac{1}{n}$.

4. $\vec{\alpha}, \vec{\beta}, \vec{\gamma}$ are three vectors satisfying the conditions

$\vec{\alpha} + \vec{\beta} + \vec{\gamma} = \vec{0}$.

If $|\vec{\alpha}| = 3$, $|\vec{\beta}| = 4$, $|\vec{\gamma}| = 5$, show that

$\vec{\alpha} \cdot \vec{\beta} + \vec{\beta} \cdot \vec{\gamma} + \vec{\gamma} \cdot \vec{\alpha} = -25$.

5. Show that curl (grad f) = 0, where $f = x^2 y + 2xy + z^2$.

6. Show that the pair of lines whose direction ratio are given by $\frac{1}{l} + \frac{1}{m} + \frac{1}{n} = 0$ and $2l - m + 2n = 0$ are perpendicular.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) A function $f(x)$ is defined by

$$f(x) = x \sin \frac{1}{x} \qquad \text{if } x \neq 0$$

$$= 0 \qquad \text{if } x = 0.$$

Prove that $f(x)$ is continuous but not differentiable at origin.

b) A variable plane is at a constant distance p from origin and meets co-ordinate axes in A, B and C. The planes are drawn through A, B and C and parallel to co-ordinate axes.

Show that locus of their point of intersection shall be

$$\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{p^2}.$$

c) Show that the series $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n}{3^n} + \frac{1}{n^3} \right)$ is convergent.

5 + 5 + 5

8. a) Evaluate $\iiint z^2 \, dx \, dy \, dz$ over the region defined by

$z \geq 0$,

$x^2 + y^2 + z^2 \leq a^2$.

b) Show that $f(x, y) = (x + y)^4 + (x - 3)^6$ has a minimum at $(3, -3)$ and $\varphi(x, y) = 4x^2 y - y^2 - 8x^4$ has a maximum at $(0, 0)$.

c) Use L'Hospital rule to evaluate $\lim\limits_{x \to 0} \dfrac{e^x + \sin x - 1}{\log(1 + x)}$.

6 + 4 + 5

9. a) If $v = \cos^{-1}\left(\dfrac{x + y}{\sqrt{x} + \sqrt{y}}\right)$, then prove that $\cos v$ is a homogeneous function fo degree $\frac{1}{2}$ and hence prove that

$$x \frac{\partial v}{\partial x} + y \frac{\partial v}{\partial y} + \frac{1}{2} \cot v = 0.$$

b) If $y = \tan^{-1} x$, show that

$(1 + x^2)\, y_{n+1} + 2nxy_n + n(n - 1)\, y_{n-1} = 0$. Hence find $(y_n)_0$.

8 + 7

10. a) State Rolle's theorem. Verify Rolle's theorem for $f(x) = \sin x \cos x$; $0 \le x \le \pi/2$ (if possible) and hence find the value of c.

b) Use Mean value theorem prove that,

$$\frac{\pi}{6} + \frac{\sqrt{3}}{15} < \sin^{-1}\left(\frac{3}{5}\right) < \frac{\pi}{6} + \frac{1}{8}.$$

c) Find, by the method of double integration, the area of the region bounded by the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$.

5 + 5 + 5

11. a) A particle moves on the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$, where t denotes time. Find the components of velocity and acceleration at time $t = 1$ in the direction $\hat{i} - 3\hat{j} + 2\hat{k}$.

b) Show that

$$\vec{A} = (6xy + z^3)\,\hat{i} + (3x^2 - z)\,\hat{j} + (3xz^2 - y)\,\hat{k}$$ is

irrotational. Find the scalar functions ϕ such that $\vec{A} = \vec{\nabla}\phi$.

c) Show that the series $\frac{2}{1^2} - \frac{3}{2^2} + \frac{4}{3^2} - \frac{5}{4^2} + \ldots\ldots$ is conditionally covergent. 5 + 5 + 5