



Name :

Roll No. :

Invigilator's Signature :

CS/B.Tech(OLD)/SEM-1/M-101/2010-11

2010-11

MATHEMATICS

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :
 $10 \times 1 = 10$

i) Degree of homogeneity of $ax^2 + 2hxy + by^2$ is

- a) 2
- b) 1
- c) 0
- d) none of these.

ii) If $u + v = x$, $uv = y$ then $\frac{\partial (x, y)}{\partial (u, v)}$ is

- a) $u - v$
- b) uv
- c) $u + v$
- d) $\frac{u}{v}$.



iii) If $f(x, y) = x^2 + y^2$, then $f_{xy}(x, y) =$

- a) 1
- b) 0
- c) 2
- d) $x + y$.

iv) The sequence $\{(-1)^n\}$ is

- a) convergent
- b) oscillatory
- c) divergent
- d) none of these.

v) The series $\sum r^n$ is divergent if

- a) $|r| > 1$
- b) $r > 1$
- c) $r \geq 1$
- d) $r > 0$.

vi) If $y = 5x^{100} + 3$, then y_{100} is equal to

- a) 100!
- b) 500
- c) 0
- d) none of these.

vii) If $y = \sin 2x + \cos 2x$, then $(y_n)_0$ is equal to

- a) $(-1)^n 2^n$
- b) 2^n
- c) 0
- d) none of these.



viii) The value of x which makes the vector $xi + 2j + 8k$ and $2i + 3j - k$ perpendicular is

- | | |
|------|--------|
| a) 0 | b) - 1 |
| c) 1 | d) 2. |

ix) If $\vec{\alpha} \cdot (\vec{\beta} \times \vec{\gamma}) = 0$, then the vectors $\vec{\alpha}, \vec{\beta}, \vec{\gamma}$ are

- | | |
|--------------|-------------------|
| a) coplaner | b) independent |
| c) collinear | d) none of these. |

x) If the radius of the sphere

$$K(x^2 + y^2 + z^2) + x + 2y - 2z = 0, \text{ is } 1, \text{ then } K =$$

- | | |
|----------------------|----------------------|
| a) ± 2 | b) $\pm \frac{2}{3}$ |
| c) $\pm \frac{3}{2}$ | d) ± 1 . |

xi) The maximum value of the directional derivative of

$$d = 2xz - y^2 \text{ at the point } (1, 3, 2) \text{ is}$$

- | | |
|----------------|------------------|
| a) $\sqrt{14}$ | b) $2\sqrt{14}$ |
| c) $2\sqrt{7}$ | d) $3\sqrt{7}$. |



xii) The value of

$$\int_0^a \int_{y=0}^{\frac{b}{a} \sqrt{a^2 - x^2}} x^3 y \, dy \, dx \text{ is}$$

- a) $\frac{b^2 a^3}{24}$ b) $\frac{b^2 a^4}{24}$
 c) $\frac{b^4 a^3}{24}$ d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. 3 × 5 = 15

2. If $W = \psi (2x - 3y, 3y - 4z, 4z - 2x)$, prove that

$$\frac{1}{2} \frac{\partial W}{\partial x} + \frac{1}{3} \frac{\partial W}{\partial y} + \frac{1}{4} \frac{\partial W}{\partial z} = 0.$$

3. Test the convergence of the series $\sum \frac{1}{\sqrt{n}} \sin \frac{1}{n}$.

4. $\vec{\alpha}, \vec{\beta}, \vec{\gamma}$ are three vectors satisfying the conditions

$$\vec{\alpha} + \vec{\beta} + \vec{\gamma} = \vec{0}.$$

If $|\vec{\alpha}| = 3, |\vec{\beta}| = 4, |\vec{\gamma}| = 5$, show that

$$\vec{\alpha} \cdot \vec{\beta} + \vec{\beta} \cdot \vec{\gamma} + \vec{\gamma} \cdot \vec{\alpha} = -25.$$

5. Show that $\text{curl} (\text{grad } f) = 0$, where $f = x^2 y + 2xy + z^2$.

6. Show that the pair of lines whose direction ratio are given by $\frac{1}{l} + \frac{1}{m} + \frac{1}{n} = 0$ and $2l - m + 2n = 0$ are perpendicular.



GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. 3 × 15 = 45

7. a) A function $f(x)$ is defined by

$$f(x) = x \sin \frac{1}{x} \quad \text{if } x \neq 0$$

$$= 0 \quad \text{if } x = 0.$$

Prove that $f(x)$ is continuous but not differentiable at origin.

- b) A variable plane is at a constant distance p from origin and meets co-ordinate axes in A, B and C . The planes are drawn through A, B and C and parallel to co-ordinate axes.

Show that locus of their point of intersection shall be

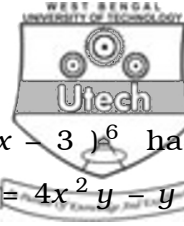
$$\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{p^2}.$$

- c) Show that the series $\sum_{n=1}^{\infty} \left(\frac{n}{3^n} + \frac{1}{n^3} \right)$ is convergent.

5 + 5 + 5

8. a) Evaluate $\iiint z^2 \, dx \, dy \, dz$ over the region defined by $z \geq 0,$

$$x^2 + y^2 + z^2 \leq a^2.$$



b) Show that $f(x, y) = (x + y)^4 + (x - 3)^6$ has a minimum at $(3, -3)$ and $\phi(x, y) = 4x^2y - y^2 - 8x^4$ has a maximum at $(0, 0)$.

c) Use L'Hospital rule to evaluate $\lim_{x \rightarrow 0} \frac{e^x + \sin x - 1}{\log(1+x)}$.
6 + 4 + 5

9. a) If $v = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, then prove that $\cos v$ is a homogeneous function of degree $\frac{1}{2}$ and hence prove that

$$x \frac{\partial v}{\partial x} + y \frac{\partial v}{\partial y} + \frac{1}{2} \cot v = 0.$$

b) If $y = \tan^{-1} x$, show that

$$(1 + x^2) y_{n+1} + 2nxy_n + n(n-1)y_{n-1} = 0. \text{ Hence find } (y_n)_0. \quad 8 + 7$$

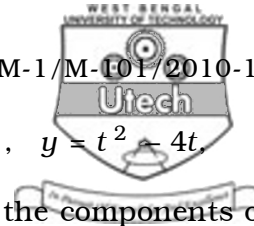
10. a) State Rolle's theorem. Verify Rolle's theorem for $f(x) = \sin x \cos x$; $0 \leq x \leq \pi/2$ (if possible) and hence find the value of c .

b) Use Mean value theorem prove that,

$$\frac{\pi}{6} + \frac{\sqrt{3}}{15} < \sin^{-1}\left(\frac{3}{5}\right) < \frac{\pi}{6} + \frac{1}{8}.$$

c) Find, by the method of double integration, the area of the region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

5 + 5 + 5



11. a) A particle moves on the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$, where t denotes time. Find the components of velocity and acceleration at time $t = 1$ in the direction $\hat{i} - 3\hat{j} + 2\hat{k}$.

b) Show that

$$\vec{A} = (6xy + z^3) \hat{i} + (3x^2 - z) \hat{j} + (3xz^2 - y) \hat{k} \text{ is}$$

irrotational. Find the scalar functions ϕ such that

$$\vec{A} = \nabla \phi.$$

c) Show that the series $\frac{2}{1^2} - \frac{3}{2^2} + \frac{4}{3^2} - \frac{5}{4^2} + \dots$ is conditionally convergent. 5 + 5 + 5

