

Name :

Roll No. :

Invigilator's Signature :

CS/B.Sc. (H), (BT)/SEM-2/BMT-204/2011

2011

BIOMATHEMATICS – II

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :

10 × 1 = 10

i) The series $\sum_{n=1}^{\infty} \frac{1}{n^3}$ is

- a) divergent
- b) convergent
- c) absolutely convergent
- d) none of these.

ii) The sequence $\{x_n\}$, where $x_n = \frac{2n-1}{n+1}$, $n \in N$ is
bounded by

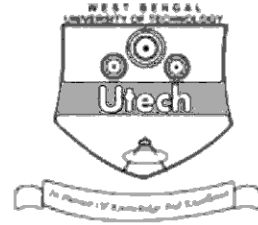
- a) 2
- b) 3
- c) $\frac{1}{2}$
- d) none of these.

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- iii) The eigenvalues of the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ are
- a) 6, 1 b) 3, 2
c) 6, 3 d) none of these.
- iv) A square matrix A is called orthogonal if
- a) $A = A^2$ b) $A^T = A^{-1}$
c) $AA^{-1} = I$ d) none of these.
- v) If two eigenvalues of $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ are 3 and 15, then the third eigenvalue is
- a) 0 b) 1
c) 4 d) none of these.
- vi) Three lines are co-planer if
- a) they are concurrent
b) a line is perpendicular to each of them
c) they are concurrent and a line is perpendicular to each of them
d) none of these.
- vii) The equation of a straight line parallel to the X-axis is given by
- a) $\frac{x-a}{1} = \frac{y-b}{1} = \frac{z-c}{1}$ b) $\frac{x-a}{0} = \frac{y-b}{1} = \frac{z-c}{1}$
c) $\frac{x-a}{0} = \frac{y-b}{0} = \frac{z-c}{1}$ d) $\frac{x-a}{1} = \frac{y-b}{0} = \frac{z-c}{0}$.



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viii) If two non-zero vectors \vec{A} and \vec{B} are parallel then

a) $\vec{A} \times \vec{B} = \vec{0}$

b) $|\vec{A} \times \vec{B}| = 1$

c) $\vec{A} \cdot \vec{B} = 0$

d) $|\vec{A}| = |\vec{B}|$

ix) The differential equation satisfying the relation

$x = A \cos (mt - \alpha)$ is

a) $\frac{dx}{dt} = 1 - x^2$

b) $\frac{d^2x}{dt^2} = -\alpha^2 x$

c) $\frac{d^2x}{dt^2} = -m^2 x$

d) $\frac{dx}{dt} = -m^2 x$

x) The order the differential equation $\left\{1 + \frac{d^2y}{dx^2}\right\}^{\frac{1}{2}} = x^2$ is

a) 1

b) 2

c) 3

d) none of these.

xi) The value of $\Gamma(3.5)$ is

a) $\frac{15\sqrt{\pi}}{8}$

b) $15\sqrt{\pi}$

c) $\frac{3\sqrt{\pi}}{4}$

d) none of these.



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xii) The complementary function of the differential equation

$$\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 9x = 0 \text{ is}$$

- a) $(c_1 + c_2t)e^{-3t}$ b) $(c_1e^{-3t} + c_2e^{-2t})$
 c) $c_1e^{-2t} + c_2e^t$ d) none of these.

xiii) The radius of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{10^n}{n} x^n \text{ is}$$

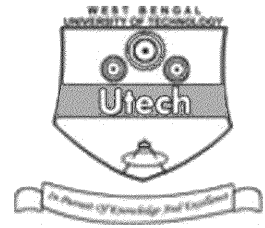
- a) 10 b) $\frac{1}{10}$
 c) 5 d) $\frac{1}{5}$.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. 3 × 5 = 15

2. a) Solve $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0$
 b) Find the particular integral of the differential equation
 $\frac{d^2y}{dx^2} + \frac{dy}{dx} = x^2 + 2x + 4.$ 3 + 2
3. a) Find the eigenvalues and eigenvectors of the
 matrix $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}.$
 b) Find the rank of the matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}.$ 2 + 3



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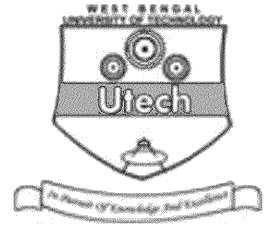
4. a) Show that any square matrix A and its transpose A^T have same eigenvalues.
- b) Find the product of the eigenvalues of the matrix $\begin{bmatrix} 2 & 3 & -2 \\ -2 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix}$. 3 + 2
5. a) If $\vec{a} = 3\vec{i} - \vec{j} + 2\vec{k}$, $\vec{b} = 2\vec{i} + \vec{j} - \vec{k}$ and $\vec{c} = \vec{i} - 2\vec{j} + 2\vec{k}$ then verify $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$.
- b) If $\vec{\alpha} = 3\vec{i} - \vec{j} + 2\vec{k}$, $\vec{\beta} = 2\vec{i} + \vec{j} - \vec{k}$ and $\vec{\gamma} = \vec{i} - 2\vec{j} + 2\vec{k}$ then show that $(\vec{\alpha} \times \vec{\beta}) \times \vec{\gamma} \neq \vec{\alpha} \times (\vec{\beta} \times \vec{\gamma})$. 2 + 3

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. 3 × 15 = 45

6. a) If two mappings $f : R \rightarrow R$ and $g : R \rightarrow R$ be defined as follows : $f(x) = x^2$, $g(x) = x - 2$, then show that $f \circ g \neq g \circ f$.
- b) Show that the mapping $f : R \rightarrow R$ defined by $f(x) = 7x + 3$, $x \in R$ is bijective.
- c) The binary operation $*$ is defined on the set of integers Z as $a * b = a + b - 2$, for all $a, b \in Z$. Show that $(Z, *)$ is a group.
- d) In a group G , if every element is its own inverse, then show that the group G is commutative. 2 + 3 + 5 + 5



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7. a) Prove that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ are co-planar.

b) Prove that the lines $\frac{x-4}{1} = \frac{y+3}{-4} = \frac{1+z}{7}$ and $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$ intersect and find the co-ordinates of their point of intersection.

c) Find the equation of the line through (1, 2, - 1) perpendicular to each of the lines $\frac{x}{1} = \frac{y}{0} = \frac{z}{-1}$ and $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$. 5 + 5 + 5

8. a) Examine the convergence of the series $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \dots \infty$

b) Discuss the convergence of the following series :

i) $\frac{1}{2\sqrt{1}} + \frac{x^2}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \frac{x^6}{5\sqrt{4}} + \dots \infty$

ii) $1 + \frac{2!}{2^2} + \frac{3!}{3^3} + \frac{4!}{4^4} + \dots \infty$

c) Discuss the conditional convergence of

$$\frac{1}{2^3} - \frac{1}{3^3}(1+2) + \frac{1}{4^3}(1+2+3) - \frac{1}{5^3}(1+2+3+4) + \dots \infty$$

3 + 4 + 4 + 4

9. a) Let $f(x) = |x|$ in $[-1, 1]$. Is Rolle's theorem applicable to $f(x)$ in $[-1, 1]$? Justify your answer.

b) In the Lagrange's mean value theorem $f(b) - f(a) = (b - a) f'(c)$ where $a < c < b$, find c if $f(x) = \sqrt{x}$, $a = 4$, $b = 9$.

c) Express the following integrals in terms of gamma function :

i) $\int_0^{\infty} e^{-x^2} dx$

ii) $\int_0^{\infty} \sqrt{x} e^{-x^3} dx.$

3 + 4 + 4 + 4

10. a) Solve any *three* of the following :

i) $(D^3 - 6D^2 + 11D - 6)y = e^{-2x} + e^{-3x}$

ii) $(D^4 - 2D^2 + 1)y = x^2 \cos x$

iii) $(D^2 + a^2)y = \sec ax$

iv) $(D - 2)^2 = 8(e^{2x} + \sin 2x + x^2).$

b) Show that the sequence $\left\{ \frac{3n-1}{2n+1} \right\}$, $n \in N$ is convergent.

3 × 4 + 3

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