BIOMATHEMATICS – II (SEMESTER - 2)

CS/B.Sc (H) (BT)/SEM-2/BMT-204/09
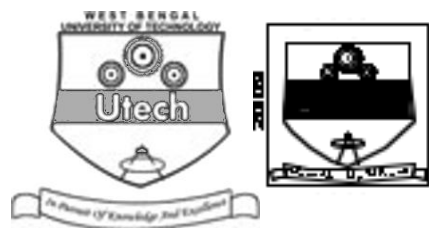
CS/B.Sc (H) (BT)/SEM-2/BMT-204/09
ENGINEERING & MANAGEMENT EXAMINATIONS, JUNE – 2009
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Time : 3 Hours]   [Full Marks : 70

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GROUP – A

(Objective Type Questions)

1. Answer any *ten* of the following : $10 \times 1 = 10$

    A.   Choose the correct alternatives for the following :

    i)   Let $f : R \to R$ be defined by $f (x) = 3x + 5$, $x \in R$, then f is

      a)   injective but not surjective   b)   surjective but injective

      c)   bijective   d)   none of these.

    ii)   The limit of the sequence $\left\{ \dfrac{n+1}{2n+1} \right\}$ $n \in N$ is

      a)   2   b) $\dfrac{1}{2}$

      c) -2 d) $-\dfrac{1}{2}$.

    iii)   The order and degree of the differential equation $\left(\dfrac{dy}{dx} \right)^2 - 5\dfrac{dy}{dx} + 6 = 0$ is

      a)   1, 2   b)   2, 1

      c)   2, 2   d)   1, 1.

    iv)   If the straight lines $\dfrac{x-1}{3} = \dfrac{y+2}{6} + \dfrac{z-3}{k}$ and $\dfrac{x+1}{2} = \dfrac{y-2}{4} + \dfrac{z+3}{6}$ are parallel, then the value of k is

      a) -7 b)   9

      c) -9 d)   7.

v)   The point where the straight line $\dfrac{x-2}{1} = \dfrac{y+3}{2} + \dfrac{z-2}{1}$ meets the xy plane is

a)   (0, – 7, 0)   b)   (1, – 7, 0)

c)   (7, – 1, 0)   d)   (1, 7, 0).

vi)   The complementary function of the differential equation $\dfrac{d^2y}{dx^2} + 3\dfrac{dy}{dx} + 2y = 0$ is

a) $Ae^{x} + Be^{2x}$ b) $Ae^{-x} + Be^{-2x}$

c) $Ae^{x} + Be^{-2x}$ d) $Ae^{-x} + Be^{2x}$.

vii)   A non-empty subset H of a group (G, o) forms a sub-group if

a) $a \in H \Rightarrow a^{-1} \in H$

b) $a, b \in H \Rightarrow aob \in H$

c) $a, b \in H \Rightarrow aob^{-1} \in H$

d) $e \in H$.

viii)   Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be defined by $f (x) = x + 1$, $x \in \mathbb{R}$ and $g (x) = 3x$, $x \in \mathbb{R}$, where $\mathbb{R}$ is the set of real numbers. Then $gof (x)$ is equal to

a) $3x + 1$ b) $3x + 3$

c) $x + 3$ d) $x + 4$.

ix)   If $\vec{\alpha} = 3\hat{i} + 4\hat{i} + \hat{k}$ and $\vec{\beta} = 2\hat{i} - \hat{j} + 4\hat{k}$, then $\vec{\alpha} . \vec{\beta}$ is equal to

a)   6   b)   2

c)   5   d)   0

x)   The order of the differential equation $\dfrac{d^2y}{dx^2} + y\dfrac{dy}{dx} + y^2 = 0$ is
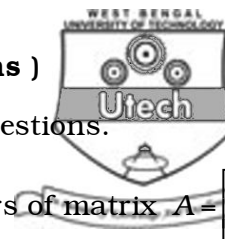
a)   2   b)   1

c)   3   d)   none of these.

B.   Write the answers very briefly :

xi)   Define equivalence relation on a set S.

xii)   Define group.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following questions.   3 × 5 = 15

2. Find the eigenvalues and the corresponding eigenvectors of matrix $A = \begin{bmatrix} -2 & 4 \\ 4 & 4 \end{bmatrix}$.

3. A relation ρ defined on the set Z by $a\rho b$ if and only if $a^2 + b^2$ is a multiple of 2. Examine if ρ is an equivalence relation.

4. Find the point where the straight line $\dfrac{x-2}{1} = \dfrac{y+3}{-1} = \dfrac{z-1}{-6}$ meets the plane $3x + y + z = 8$.

5. Find the equations of the straight line passing through the point $(-1, 1, -3)$ and perpendicular to the straight line $\dfrac{x-3}{-2} = \dfrac{y+1}{3} = \dfrac{z-2}{-4}$.

6. Find the rank of the matrix $\begin{pmatrix} 1 & 1 & 2 \\ 2 & 4 & 4 \\ 3 & 3 & 7 \end{pmatrix}$.

7. Show that the mapping $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 2x + 3$ $x \in \mathbb{R}$ is bijective.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following questions.   3 × 15 = 45

8. a) If $f(x) = x$, when $x < 1$

   $= 2 - x$, when $x \geq 1$

   show that $f^{l}(1)$ does not exist.   5

   b) Show that Rolle's theorem is not applicable for the function $f(x) = |x|$ in $-1 \leq x \leq 1$.   5

   c) Using Gamma function, evaluate $\displaystyle\int_{0}^{\infty} e^{-x^2}\, dx$.   5

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9. a) Show that the sequence $\{x_n\}$, where $x_n = \dfrac{n+1}{4n+1}$, $n \in N$ is of convergent.

   b) Find the radius of convergence and interval of convergence the power series $\displaystyle\sum_{n=1}^{\infty} a_n\, x^n$, where a_n is given by $a_n = \dfrac{1}{\log(n+1)}$.

   c) Compute $f(x)$ in Fourier series in the interval $-2 < x < 2$ if

   $f(x)\ = 0$, for $-2 < x < 1$

   　　　$= 1$, for $0 < x < 2$　　　　　　　　　　　　　　　15

10. a) Prove that the set D of all odd integers forms a group with respect to the composition $*$ defined by $a * b = a + b - 1$ for all $a, b \in D$.　　　5

   b) Define the sub-group of a group G and give an example. State the necessary and sufficient condition for a non-empty subset S of a group G to be a sub-group of G.

   　　　　　　　　　　　　　　　　　　　　　　　　5

   c) Define a cyclic group and find the generators of the cyclic group $G = \{1, -1, i, -i\}$ with respect to multiplication, where $i^2 = -1$.　　　5

11. a) Solve : $\dfrac{d^2y}{dx^2} + a^2 y = \sec ax$.　　　　　　　　　　5

   b) Solve : $\dfrac{d^2y}{dx^2} - 7\dfrac{dy}{dx} + 6y = (x-2)e^x$.　　　　　5

   c) Solve : $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + y = e^{-x} \log x$.　　　　　5

12. a) Test the convergence of the series $\dfrac{1}{2^3} - \dfrac{(1+2)}{3^3} + \dfrac{(1+2+3)}{4^3} - \dfrac{(1+2+3+4)}{5^3} + \ldots\ldots$.　　5

   b) Examine the nature of the series $\left(\dfrac{2^2}{1^2} - \dfrac{2}{1}\right)^{-1} + \left(\dfrac{3^3}{2^3} - \dfrac{3}{2}\right)^{-1} + \left(\dfrac{4^4}{3^4} - \dfrac{4}{3}\right)^{-3} + \ldots..$　　5

   c) Test the convergence of the series $\dfrac{1}{1.2.3} + \dfrac{3}{2.3.4} + \dfrac{5}{3.4.5} + \ldots..$　　5

END