Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/BNS/SEM-4/BNS-402/2012

2012

APPLIED MATHEMATICS – IV

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : 10 × 1 = 10

i) The value of $\begin{vmatrix} 1996 & 2004 & 2010 \\ 1997 & 2005 & 2011 \\ 1998 & 2006 & 2012 \end{vmatrix}$ is

a) 1

b) 0

c) 2013

d) none of these.

ii) The matrix $A = \begin{pmatrix} 0 & 2 & -7 \\ -2 & 0 & 9 \\ 7 & -9 & 0 \end{pmatrix}$ is

a) symmetric   b) skew-symmetric

c) hermitian   d) none of these.

iii) The rank of the matrix $A = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$ is

a) 1   b) 2

c) c d) none of these.

iv) The equation $x + y + z = 0$ has

a) infinite number of solutions

b) no solutions

c) unique solutions

d) two solutions.

v) The eigenvalues of the matrix A are 2 & 3. Then the eigenvalues of A^2 are

a) 2, 3   b) 4, 3

c) 4, 9   d) 2, 9.

vi) Which of the following is not true (the notations have their usual meanings) ?

a) $\Delta = E - 1$ b) $\Delta . \nabla = \Delta - \nabla$

c) $\frac{\Delta}{\nabla} = \Delta + \nabla$ d) $\Delta = 1 - E^{1}$.

vii) $\Delta^2 e^x$ is equal to (where $h = 1$)

a) $(e - 1)^2 e^x$ b) $(e - 1) e^x$

c) $e^{2x} (e - 1)$ d) $e^{2x + 1}$.

viii) In Simpson's $\frac{1}{3}$rd rule of finding $\int_a^b f(x)\,dx$, $f(x)$ is approximated by

a) line segment b) parabola

c) circular sector d) parts of ellipse.

ix) Lagrange interpolation formula is used for

a) equal interval

b) unequal interal

c) both equal & unequal intervals

d) none of these.

x) The mean of Binomial distribution Bin(np) (where n & p are the number of trials and probability of success) is

a) $\frac{n}{p}$ b) 0

c) np d) 1.

xi) If X is normally distributed with zero mean and unit variance, then the expectation of X is

a) 1 b) 2

c) 8 d) 0.

xii) $b_{yx} \times b_{xy}$ (where b_{yx} , b_{xy} and r are regression and correlation coefficients) is

a) r b) r^2

c) $\frac{1}{r}$ d) $\frac{1}{r^2}$.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Reduce the following matrix into its normal form and hence find its rank :

$$A = \begin{pmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{pmatrix}.$$

3. Using the partition method, find the inverse of

$$\begin{pmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{pmatrix}$$

4. Find the cubic polynomial which takes the following values :

x :	0	1	2	3
$f(x)$:	1	2	1	10

5. Obtain the function whose first difference is $9x^2 + 11x + 5$.

6. A hospital switchboard receives on an average 4 emergency calls in a five-minute interval. What is the probability that there are (i) at most two emergency calls in a five-minute interval, (ii) exactly 3 emergency calls in a five-minute interval ?

7. Obtain the mean and median for the following frequency distribution :

x	1	2	3	4	5	6	7	8	9
f	8	10	11	16	20	25	15	9	6

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

8. a) Diagonalise the matrix $\begin{pmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{pmatrix}$.

b) Reduce the quadratic form

$3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$ to the canonical form.

c) Verify Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$ and hence find its inverse. 5 + 5 + 5

9. a) Find a positive root of $x^2 + 2x - 2 = 0$ by Newton-Raphson method correct to four decimal places.

b) Evaluate $\int_0^6 \frac{dx}{1 + x^2}$ by using Simpson's one-third rule correct up to two places of decimal taking seven points.

c) Solve the following set of simultaneously linear equations by Gauss-elimination method :

$$2x_1 + 3x_2 + x_3 = 9$$

$$x_1 + 2x_2 + 3x_3 = 6$$

$$3x_1 + x_2 + 2x_3 = 8$$

correct up to 3-significant figures. 5 + 5 + 5

10. a) Investigate the values of λ and μ so that the equations $2x + 3y + 5z = 9$, $7x + 3y - 2z = 8$, $2x + 3y + \lambda z = \mu$ have (i) no solution, (ii) a unique solution and (iii) an infinite number of solutions.

b) Prove that value of r (correlation coefficient) lies between -1 and 1 *i.e.*, $-1 \leq r \leq 1$.

c) Express $\dfrac{x^2 + x + 1}{(x-1)(x-2)(x-3)(x-4)}$ as partial fraction using Lagrangian interpolation formula.

5 + 5 + 5

11. a) The probability density function of a continuous distribution is given by $f(x) = \frac{3}{4} x (2 - x)$, $0 < x \; 2$. Compute the mean and variance.

b) The joint probability function of two discrete random variables X and Y is given by $f(x, y) = c(2x + y)$ where x and y can assume all integers such that $0 \leq x \leq 2$, $0 \leq y \leq 3$ and $f(x, y) = 0$ otherwise.

i) Find the value of the constant c

ii) Find $p(X = 2, Y = 1)$

iii) Find $p(X \geq 1, Y \leq 2)$

c) The length of bolts produced by a machine is normally distributed with mean 4 and S.D. 0·5. A bolt is defective if its length does not lie in the interval (3·8, 4·3). Find the percentage of defective bolts produced by the machine.

[Given $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0 \cdot 6} e^{-t^2/2} \, dt = 0 \cdot 7257$ and

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0 \cdot 4} e^{-t^2/2} \, dt = 0 \cdot 6554 \text{]}$$

12. a) The skewness γ of a random variable X is defined by $\gamma = \frac{1}{\sigma^3} E\left([X - \mu]^3 \right)$.

Show that for a symmetric distribution (whose third central moment exists) the skewness is zero.

b) Solve by Jacobi's iteration method, the system of equations :

$20x + y - 2z = 17, \;\; 3x + 20y - z = -18,$

$2x - 3y + 20z = 25.$ 7 + 8