



Name :

Roll No. :

Invigilator's Signature :

**CS/BNS/SEM-4/BNS-402/2012
2012**

APPLIED MATHEMATICS – IV

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

**GROUP – A
(Multiple Choice Type Questions)**

1. Choose the correct alternatives for any *ten* of the following :

$$10 \times 1 = 10$$

i) The value of

1996	2004	2010
1997	2005	2011
1998	2006	2012

 is

- a) 1
- b) 0
- c) 2013
- d) none of these.



ii) The matrix $A = \begin{pmatrix} 0 & 2 & -7 \\ -2 & 0 & 9 \\ 7 & -9 & 0 \end{pmatrix}$ is

- a) symmetric
- b) skew-symmetric
- c) hermitian
- d) none of these.

iii) The rank of the matrix $A = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$ is

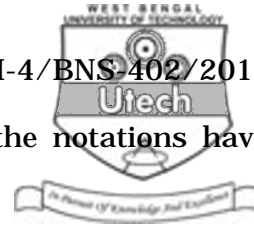
- a) 1
- b) 2
- c) c
- d) none of these.

iv) The equation $x + y + z = 0$ has

- a) infinite number of solutions
- b) no solutions
- c) unique solutions
- d) two solutions.

v) The eigenvalues of the matrix A are 2 & 3. Then the eigenvalues of A^2 are

- a) 2, 3
- b) 4, 3
- c) 4, 9
- d) 2, 9.



vi) Which of the following is not true (the notations have their usual meanings) ?

- a) $\Delta = E - 1$ b) $\Delta \cdot \nabla = \Delta - \nabla$
 c) $\frac{\Delta}{\nabla} = \Delta + \nabla$ d) $\Delta = 1 - E^1$.

vii) $\Delta^2 e^x$ is equal to (where $h = 1$)

- a) $(e - 1)^2 e^x$ b) $(e - 1) e^x$
 c) $e^{2x} (e - 1)$ d) e^{2x+1} .

viii) In Simpson's $\frac{1}{3}$ rd rule of finding $\int_a^b f(x) dx$, $f(x)$ is

approximated by

- a) line segment b) parabola
 c) circular sector d) parts of ellipse.

ix) Lagrange interpolation formula is used for

- a) equal interval
 b) unequal interval
 c) both equal & unequal intervals
 d) none of these.

- ### GROUP – B

Answer any *three* of the following. $3 \times 5 = 15$

- $$A = \begin{pmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{pmatrix}.$$



3. Using the partition method, find the inverse of

$$\begin{pmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{pmatrix}$$

4. Find the cubic polynomial which takes the following values :

x :	0	1	2	3
f (x) :	1	2	1	10

5. Obtain the function whose first difference is $9x^2 + 11x + 5$.
6. A hospital switchboard receives on an average 4 emergency calls in a five-minute interval. What is the probability that there are (i) at most two emergency calls in a five-minute interval, (ii) exactly 3 emergency calls in a five-minute interval ?
7. Obtain the mean and median for the following frequency distribution :

x	1	2	3	4	5	6	7	8	9
f	8	10	11	16	20	25	15	9	6



GROUP - C

(Long Answer Type Questions)

Answer any *three* of the following.

$3 \times 15 = 45$

8. a) Diagonalise the matrix $\begin{pmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{pmatrix}$.

b) Reduce the quadratic form

$3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$ to the canonical form.

c) Verify Cayley-Hamilton theorem for the matrix

$A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$ and hence find its inverse. $5 + 5 + 5$

9. a) Find a positive root of $x^2 + 2x - 2 = 0$ by Newton-Raphson method correct to four decimal places.

b) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using Simpson's one-third rule

correct up to two places of decimal taking seven points.

c) Solve the following set of simultaneously linear equations by Gauss-elimination method :

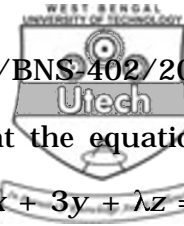
$$2x_1 + 3x_2 + x_3 = 9$$

$$x_1 + 2x_2 + 3x_3 = 6$$

$$3x_1 + x_2 + 2x_3 = 8$$

correct up to 3-significant figures.

$5 + 5 + 5$



10. a) Investigate the values of λ and μ so that the equations $2x + 3y + 5z = 9$, $7x + 3y - 2z = 8$, $2x + 3y + \lambda z = \mu$ have (i) no solution, (ii) a unique solution and (iii) an infinite number of solutions.

- b) Prove that value of r (correlation coefficient) lies between -1 and 1 i.e., $-1 \leq r \leq 1$.

- c) Express $\frac{x^2 + x + 1}{(x-1)(x-2)(x-3)(x-4)}$ as partial fraction using Lagrangian interpolation formula.

$$5 + 5 + 5$$

11. a) The probability density function of a continuous distribution is given by $f(x) = \frac{3}{4}x(2-x)$, $0 < x < 2$.

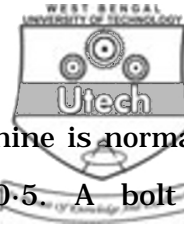
Compute the mean and variance.

- b) The joint probability function of two discrete random variables X and Y is given by $f(x, y) = c(2x + y)$ where x and y can assume all integers such that $0 \leq x \leq 2$, $0 \leq y \leq 3$ and $f(x, y) = 0$ otherwise.

i) Find the value of the constant c

ii) Find $p(X = 2, Y = 1)$

iii) Find $p(X \geq 1, Y \leq 2)$



- c) The length of bolts produced by a machine is normally distributed with mean 4 and S.D. 0.5. A bolt is defective if its length does not lie in the interval (3.8, 4.3). Find the percentage of defective bolts produced by the machine.

$$\left[\text{Given } \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0.6} e^{-t^2/2} dt = 0.7257 \text{ and} \right.$$

$$\left. \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0.4} e^{-t^2/2} dt = 0.6554 \right]$$

12. a) The skewness γ of a random variable X is defined by $\gamma = \frac{1}{\sigma^3} E([X - \mu]^3)$.

Show that for a symmetric distribution (whose third central moment exists) the skewness is zero.

- b) Solve by Jacobi's iteration method, the system of equations :

$$20x + y - 2z = 17, \quad 3x + 20y - z = -18,$$

$$2x - 3y + 20z = 25.$$

7 + 8
