



Name :

Roll No. :

Invigilator's Signature :

CS/BNS/SEM-2/BNS-201/2012

2012

APPLIED MATHEMATICS - II

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :

$$10 \times 1 = 10$$

i) The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if

a) $p \geq 1$

b) $p > 1$

c) $p < 1$

d) $p \leq 1$.

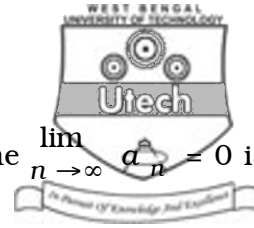
ii) The series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \dots$ is

a) convergent

b) divergent

c) oscillatory

d) none of these.



iii) If $\sum a_n$ is convergent series, then the $\lim_{n \rightarrow \infty} a_n = 0$ is

- a) necessary condition
- b) sufficient condition
- c) neither necessary nor sufficient condition
- d) necessary as well as sufficient condition.

iv) Relation between beta and gamma function is

- a) $B(m, n) = \frac{\Gamma(m)}{\Gamma(n) \Gamma(m+n)}$
- b) $B(m, n) = \frac{\Gamma(n)}{\Gamma(m) \Gamma(m+n)}$
- c) $B(m, n) = \frac{\Gamma(m) \Gamma(n)}{\Gamma(m+n)}$
- d) none of these.

v) The error function or the probability integral is defined as

- a) $erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$
- b) $erf(x) = \frac{1}{\sqrt{\pi}} \int_0^x e^{-t} dt$
- c) $erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^3} dt$
- d) none of these.



vi) The value of triple integral $\int_a^b \int_c^d \int_e^f dx dy dz$

- a) $a + b + c + d + e + f$
- b) $abcdef$
- c) $(b - a)(d - c)(f - e)$
- d) $(a + b)(c + d)(e + f)$.

vii) If c is the circle $x^2 + y^2 = 4$, then $\int_c x^2 dx$ is

- a) 0
- b) $\frac{1}{3}$
- c) 3
- d) 1.

viii) The moment of inertia of thin uniform rod of mass M and length $2a$ about an axis perpendicular to the rod at its centre is

- a) $\frac{Ma^2}{3}$
- b) $\frac{Ma^2}{2}$
- c) Ma^2
- d) $\frac{Ma^2}{4}$.

ix) The value of $\int_0^1 \frac{dx}{1+x}$ by Simpson's $\frac{1}{3}$ rd rule is

- a) 0.96315
- b) 0.63915
- c) 0.69315
- d) none of these.

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GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following.

3 × 5 = 15

2. Obtain the Fourier series for $f(x) = e^{-x}$ in the interval $0 < x < 2\pi$.
3. The velocity v (km/min) of a moped which starts from rest, is given at fixed intervals of time t (min) as follows :

x	2	4	6	8	10	12	14	16	18	20
y	10	18	25	29	32	20	11	5	2	0

Estimate approximately the distance covered in 20 minutes by Simpson's one-third rule.

4. Evaluate :

$$\int_0^{\infty} \int_0^{\infty} e^{-(x^2 + y^2)} dx dy$$

by changing to polar coordinates. Hence, show that

$$\int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$$

5. The sides of a spherical triangle ABC are all quadrants and x, y, z are the arcs joining any point within the triangle to the angular points. Prove that

$$\cos^2 x + \cos^2 y + \cos^2 z = 1.$$



6. State Cauchy's root test and apply it to discuss the

convergence of
$$\sum_{n=1}^{\infty} \frac{n^{n^2}}{(1+n)^{n^2}}$$

7. If δ be the length of the arc from the vertex of an isosceles triangle dividing the base into segments α and β , then prove that $\tan \frac{\alpha}{2} \tan \frac{\beta}{2} = \tan \left(\frac{\alpha + \delta}{2} \right) \tan \left(\frac{\alpha - \delta}{2} \right)$.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

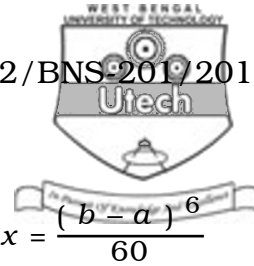
8. a) Prove that
$$\int_0^1 \left(\sum_{n=2}^{\infty} \frac{x^n}{n^2} \right) dx = \sum_{n=1}^{\infty} \frac{1}{n^2 (n+1)}.$$

b) Find the Fourier sine transform of $e^{-|x|}$. Hence show

that
$$\int_0^{\infty} \frac{x \sin mx}{1+x^2} dx = \frac{\pi e^{-m}}{2}, m > 0.$$

c) If in a spherical ΔABC , α, β be the arcs drawn from right angle C respectively perpendicular to and bisecting the hypotenuse c , show that

$$\sin^2 \frac{1}{2} C (1 + \sin^2 \alpha) = \sin^2 \beta.$$
 $5 + 5 + 5$



9. a) Show that $\int_a^b (x-a)^3 (b-x)^2 dx = \frac{(b-a)^6}{60}$
- b) Show that $\int_0^\infty e^{-4x} x^{3/2} dx = \frac{3}{128} \sqrt{\pi}$.
- c) Show that the Fourier series corresponding to $f(x) = x^2$ in $-\pi \leq x \leq \pi$ is $\frac{\pi^2}{3} + 4 \sum_{n=1}^{\infty} (-1)^n \frac{\cos nx}{n^2}$.
- 5 + 5 + 5

10. a) A solid of revolution is formed by rotating about the x -axis, the area between the x -axis, the lines $x = 0$ and $x = 1$ and a curve through the points with the following co-ordinates :

$x :$	0.00	0.25	0.50	0.75	1.00
$y :$	1.0000	0.9896	0.9589	0.9089	0.8415

Estimate the volume of the solid formed using Simpson's rule.



- b) Find the Moment of Inertia of the area bounded by the curve $r^2 = a^2 \cos 2\theta$ about its axis. 7 + 8

11. a) Prove that $B\left(m, \frac{1}{2}\right) = 2^{2m-1} B(m, m)$.

- b) Find the area of the portion of the cylinder $x^2 + z^2 = 4$ lying inside the cylinder $x^2 + y^2 = 4$.

- c) Given $C = 69^\circ 25'$, $A = 54^\circ 55'$, $C = 90^\circ$. Solve the triangle. 5 + 5 + 5

12. a) Evaluate $\int \int r \sin \theta \, dr d\theta$ over the area of the cardioid

$$r = a(1 + \cos \theta) \text{ above the initial line.}$$

- b) Show that the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16}{3} a^2$.

- c) Find the volume of the tetrahedron bounded by the coordinate planes and the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.

5 + 5 + 5

