Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS/BNS/SEM-2/BNS-201/2012

2012

APPLIED MATHEMATICS - II

Time Allotted : 3 Hours Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if

a) $p \geq 1$ b) $p > 1$

c) $p < 1$ d) $p \leq 1$.

ii) The series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \ldots\ldots\ldots$ is

a) convergent b) divergent

c) oscillatory d) none of these.

iii) If $\sum a_n$ is convergent series, then the $\lim_{n \to \infty} a_n = 0$ is

a) necessary condition

b) sufficient condition

c) neither necessary nor sufficient conditon

d) necessary as well as sufficient condition.

iv) Relation between beta and gamma functiona is

a) $B(m, n) = \dfrac{\Gamma(m)}{\Gamma(n)\,\Gamma(m+n)}$

b) $B(m, n) = \dfrac{\Gamma(n)}{\Gamma(m)\,\Gamma(m+n)}$

c) $B(m, n) = \dfrac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$

d) none of these.

v) The error function or the probability integral is defined as

a) $erf(x) = \dfrac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt$

b) $erf(x) = \dfrac{1}{\sqrt{\pi}} \int_0^x e^{-t}\, dt$

c) $erf(x) = \dfrac{2}{\sqrt{\pi}} \int_0^x e^{-t^3}\, dt$

d) none of these.

vi) The value of triple integral $\int_a^b \int_c^d \int_e^f dx\,dy\,dz$

a) $a + b + c + d + e + f$

b) $abcdef$

c) $(b - a)(d - c)(f - e)$

d) $(a + b)(c + d)(e + f)$.

vii) If c is the circle $x^2 + y^2 = 4$, then $\int_c x^2\,dx$ is

a) 0 b) $\frac{1}{3}$

c) 3 d) 1.

viii) The moment of inertia of thin uniform rod of mass M and length $2a$ about an axis perpendicular to the rod at its centre is

a) $\frac{Ma^2}{3}$ b) $\frac{Ma^2}{2}$

c) Ma^2 d) $\frac{Ma^2}{4}$.

ix) The value of $\int_0^1 \frac{dx}{1 + x}$ by Simpson's $\frac{1}{3}$rd rule is

a) 0·96315 b) 0·63915

c) 0·69315 d) none of these.

x) The period of the function $f(x) = \sin nx$ is

a) $2\_$ b) $2n\_$

c) $\frac{2\pi}{n}$ d) 4π.

xi) Parseval's identity states that $\frac{1}{L}\int_{-L}^{L} (f(x))^2 \, dx$ is equal to

a) $\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n + b_n)$

b) $\frac{{a_0}^2}{2} + \sum_{n=1}^{\infty} ({a_n}^2 + {b_n}^2)$

c) $\sum_{n=1}^{\infty} ({a_n}^2 + {b_n}^2)$

d) none of these.

xii) Simpson's one third rule gives an exact value of the integral if integrant $f(x)$ is a polynomial of degree at most

a) two b) one

c) three d) four.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Obtain the Fourier series for $f(x) = e^{-x}$ in the interval $0 < x < 2\pi$.

3. The velocity v (km/min) of a moped which starts from rest, is given at fixed intervals of time t (min) as follows :

x	2	4	6	8	10	12	14	16	18	20
y	10	18	25	29	32	20	11	5	2	0

Estimate approximately the distance covered in 20 minutes by Simpson's one-third rule.

4. Evaluate :

$$\int_0^{\infty} \int_0^{\infty} e^{-(x^2 + y^2)} \, dx dy$$

by changing to polar coordinates. Hence, show that

$$\int_0^{\infty} e^{-x^2} \, dx = \frac{\sqrt{\pi}}{2}$$

5. The sides of a spherical triangle ABC are all quadrants and x, y, z are the arcs joining any point within the traingle to the angular points. Prove that

$$\cos^2 x + \cos^2 y + \cos^2 z = 1.$$

6. State Cauchy's root test and apply it to discuss the convergence of $\sum_{1}^{\infty} \frac{n^{n^2}}{(1+n)^{n^2}}$

7. If δ be the length of the arc from the vertex of an isosceles triangle dividing the base into segments α and β, then prove that $\tan\frac{\alpha}{2}\tan\frac{\beta}{2} = \tan\left(\frac{\alpha+\delta}{2}\right)\tan\left(\frac{\alpha-\delta}{2}\right)$.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

8. a) Prove that $\int_0^1 \left(\sum \frac{x^n}{n^2}\right) dx = \sum_{n=1}^{\infty} \frac{1}{n^2(n+1)}$.

b) Find the Fourier sine transform of $e^{-|x|}$. Hence show that $\int_0^{\infty} \frac{x \sin mx}{1+x^2} dx = \frac{\pi e^{-m}}{2}$, $m > 0$.

c) If in a spherical ΔABC, α, β be the arcs drawn from right angle C respectively perpendicular to and bisecting the hypotenuse c, show that $\sin^2\frac{1}{2}C(1+\sin^2\alpha) = \sin^2\beta$. 5 + 5 + 5

9. a) Show that $\int_a^b (x-a)^3 (b-x)^2 \, dx = \frac{(b-a)^6}{60}$

b) Show that $\int_0^\infty e^{-4x} x^{3/2} \, dx = \frac{3}{128}\sqrt{\pi}$.

c) Show that the Fourier series coresponding to $f(x) = x^2$ in $-\pi \le x \le \pi$ is $\frac{\pi^2}{3} + 4 \sum_{n=1}^{\infty} (-1)^n \frac{\cos nx}{n^2}$.

5 + 5 + 5

10. a) A solid of revolution is formed by rotating about the x-axis, the area between the x-axis, the lines $x = 0$ and $x = 1$ and a curve through the points with the following co-ordinates :

x :	0·00	0·25	0·50	0·75	1·00
y :	1·0000	0·9896	0·9589	0·9089	0·8415

Estimate the volume of the solid formed using Simpson's rule.

b) Find the Moment of Inertia of the area bounded by the curve $r^2 = a^2 \cos 2\theta$ about its axis. 7 + 8

11. a) Prove that $B\left(m, \frac{1}{2}\right) = 2^{2m-1} B(m, m)$.

b) Find the area of the portion of the cylinder $x^2 + z^2 = 4$ lying inside the cylinder $x^2 + y^2 = 4$.

c) Given $C = 69^\circ 25'$, $A = 54^\circ 55'$, $C = 90^\circ$. Solve the triangle. 5 + 5 + 5

12. a) Evaluate $\int\int r \sin\theta \, dr d\theta$ over the area of the cardioid

$r = a(1 + \cos\theta)$ above the initial line.

b) Show that the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16}{3} a^2$.

c) Find the volume of the tetrahedron bounded by the coordinate planes and the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.

5 + 5 + 5